

Krantz, *How to Teach Mathematics*

Reading notes by Daniel R. Collins

Preface to 1st Ed.

- **Primary problem: Considers calculus lowest level. For us: Highest level. “My examples... ranging from calculus to real analysis and beyond” (p. xix).**
- Prescriptive (p. xx), time-tested techniques. Not expecting reform/experimental strategies.

Preface to 2nd Ed.

- Traditionalist
- *On Proofs: Suggest informal; notes sophomore transitions course for majors. I disagree; want to share idea of proof by College Algebra (connect to high school geometry). Issue in our major/assessment that we don't have transitions course.*
- Teaching: Personal; no one “correct” answer.

Chapter 1: Guiding Principles

1.1 Respect

- “Preparation is the core of effective teaching” (p. 2).
- Treat questions with respect.

1.2 Prepare

- “Utmost importance” (p. 4).
- Confident to answer questions on the fly.
- Debrief after class.

1.3 Speak Up

- “Find a way to fill the room with yourself” (p. 5).
- Not a ham, showoff, or joke-teller.

1.4 Lectures

- **Pro-lecture; “Lectures have been used to good effect for more than 3000 years”. (p. 7)**
- Compare to self-help, motivational, religious speakers.
- Reformers argue for group work, about half material covered.
- DRC: Arguing that all learning is self-discovery (we cherish those memories) is a violation of the very strength of language itself; to share experience efficiently.

1.5 Questions

- Turning bad questions into good ones on the fly [DRC: jujutsu-ing student response/question; *what question would this have been correct for?*; inverse challenge].
- Saying “I don’t know”, get back, when necessary. “If there is any circumstance in which honesty is the best policy, this is it.” (p. 14)

1.6 Time

- “Several aspects of teaching that require time management skills” (p. 14). [DRC: I might argue this is one of *the* top priorities of the discipline.]
- **Argues for lecture in approx. 5-minute chunks at end so it can be flexed to fit in class time [DRC: I find this feasible in regards to adjusting exercises, but not lecture presentations].**
- Make sure tests fit allotted time.

1.7 Applications

- Calculus-specific discussion (“a matter of personal taste”, p. 17). [DRC: Arguably we don’t have time for rigorous applications in our classes; more formalized syllabus/curriculum. Maybe one extra class meeting in my statistics/college algebra courses; e.g., find final exam score needed to get specified grade in course].
- Testing on applications is a very hard problem.
- Green’s theorem, planimeter, storming of School of Architecture (p. 19).

1.8 Moore Method

- Pure self-discovery, harsh critiques by instructor [DRC: compare to art school]. Forbidden from outside texts, resources, or working together.
- Obviously beyond unacceptable for our students (equity).
- “Practiced in all his classes, from the most elementary undergraduate...”; but other sources asserts for “advanced”, “graduate” courses (Wikipedia, quote by F.B. Jones).

Chapter 2: Practical Matters

2.1 Voice

- “We must *fill the room.*” (p. 23).
- Lowering voice, pausing; not monotone. [DRC: Clearmountain pause.]
- Not silly, lose dignity, etc. (Hollywood acting coach, bad joke, p. 24).
- Don’t rush; fit time appropriately [DRC: Saw general talking, very slow; projects authority.]
- Make it look fairly easy (counter to seeing instructor wrestle as example).

2.2 Eye Contact

- As good conversationalist; hold the eyes.
- DRC: *Listening is important.*

2.3 Blackboard Technique

- Note discussion still in terms of “blackboard” not “whiteboard” [DRC: Our department still keeps blackboards in our wing, no computers/projectors].
- Write clearly, in lines. Box off board in separate sections clearly (“To repeat: If necessary, *actually divide the blackboard into boxes.*”; p. 27). Advantage to being left-handed in that we don’t block writing as it appears. Try to think ahead: Material to be kept through period should be placed on edge (or overhead slide; DRC: as I do now, recent addition?). Practicing drawing figures by hand, esp. 3D. *Do not erase* until necessary; then, carefully and completely. Okay to take time; stand back and consider (students catch up; breath). Write as much as possible. Don’t re-use problem description; re-write. (?) *Don’t fall into trap* of rushing verbally.
- “Writing material neatly and slowly is a subtle way of telling students that this material is important” (p. 30). [DRC: Used to argue the same, but had time-management issues. Now I use slides; sure to be clear, save time, communicate importance that I took time to prepare them.]

2.4 Homework

- Homework should not be too short, too long, etc.; touches on all most important topics.
- “*Make sure that at least some of the homework problems are graded.*” (p. 30). [DRC: Couldn’t do that, drove me crazy for years due to crazy student writing, copying, combativeness over grades.] “If the last suggestion will not work for you, then you can give weekly quizzes that you yourself will grade.” (p. 31) [DRC: Online auto-graded.] New internet homework platforms. [DRC: Some success; some technical problems, connection, entering notation, etc.]
- Encouraging group work (e.g., Asian students).

2.5 Office Hours

- DRC: Some people look for reasons/motivations to get students to office hour. I feel it’s a resource of last resort; indicates some problem with course; never used it myself (or friends). Prefer email (incl. weekends). Question: Response if one weak student hogs office hour every day? (StackExchange)

2.6 Designing a Course

- “Many of us will never have the privilege of actually designing a course.”

2.7 Handouts

- After a poor lecture, “I can salve my conscience by writing up a handout... In my heart of hearts, however, I know that what I *should* do is strive to give better classes.”

2.8 Teaching Evaluations

- Student evaluations have some value in aggregate. [DRC: I’ve used them in the past to improve my course; e.g., “instructor keeps me informed of progress”, started using Blackboard Grade Center. KCC online evaluations collapse; e.g., ~3 responses per classes; statistically useless; students routinely say the login doesn’t work; I can remember last time I received report].
- DRC: Video documentation of course. Today can use laptop & portable camera.

2.9 Exams

- “Handwritten exams... are good. Multiple choice... not so good”. [DRC: Debated in some departments. Half-and-half would be excellent; customary on Common Core, PARCC, SATs.]
- Straightforward and transparent about subject matter (95%). “Your exams are one of your most important tools for communicating with your class.” [DRC, from some other book: Students awareness heightened on test; a learning opportunity. I sometimes give hints, i.e., “think back to this”, or “same as homework”]. “*Make sure that the questions you ask elicit the basic information that you seek.*” Important to work the test problems in advance [DRC: I do this for length, and partial-scoring rubric.]
- Short answer vs. multiple choice: Which tests the students on what they should really be learning? (The former, more so.) “Hand-graded exams are better.”
- Practice exam useful; review sessions not so much. Possible take-home for corrective credit.
- An exam you can do in 20 minutes is about right for 50 minutes given to freshmen.
- Display the point value clearly on each exam question. Consider breaking up large problems into multiple steps. [DRC: I do this in places in statistics.]
- Record statistics of results (“as a guide in setting up your grading curve”).
- Hand back exams as soon as possible after exam is given [DRC: I always do this in next class period; many students are surprised]. Do not do exam problem in class time. Meet privately for any questions.

2.10 Grading

- Have grading be part of public record on first day on syllabus. Stick to that. Show respect to student inquiries after exams. A formal system (not “subjective”) heads off complaints/grievances later. Krantz gives course “A” for any final exam “A”, with debate from other students and faculty. [DRC: I wouldn’t do this; compare to our standards of must-pass both.]
- Don’t get reputation as instructor who will negotiate grades (85% of students lined up outside office; unpleasant and degrading to both sides). “*Make students read your (posted) solution before you agree to talk about the grading of the problem.*”
- Possible mandatory department grading scheme/curve. [DRC: Loathe grading to a curve; can easily format tests so average is almost always near 75%. Shocked by how standard this still is in academia; StackExchange; SW’s partner at BU, to standardize across sections.]
- Absurdity of students (commonly) asking for raised grade after semester ends. Example of a Dean routinely altering grades by fiat after meeting student.

2.11 The Syllabus

- “For most courses, a syllabus of one or two pages is more than sufficient.” (Possible exception: several sections being coordinated on same daily schedule.) Outline form, not paragraphs.
- Keep a course diary of decisions, makeup test dates, fill-in travel dates, etc. [DRC: I do this in standard text file.]

2.12 Choosing a Textbook

- DRC: We have books chosen at department level, officially. (Krantz mentions elsewhere this is the one time he was officially disciplined.)
- Look at it from student’s perspective. Poor text is suffered throughout semester. Example: Dispute over whether we say function is discontinuous at point where it is undefined. [DRC: Common on StackExchange!] Use same conventions, notations, etc. as text in class (else: student confusion). If you don’t like those conventions, don’t use that book. Safest policy is to consult someone who has used it; at least read a sample chapter.
- Pick one book and stick with it (not dipping in multiple books; not a literature/philosophy class). Students can consult earlier on book for help with same notation & definitions. Pick something for your level of students.
- Anti-Saxon Math. Too much hand-holding (1-page lesson chunks).
- Too easy to digitally click for hints/answer: students skip the work. Some barrier to finding answer (at back) is good.
- Consider cost to students.

2.13 Large Lectures

- Practice using microphone. Welcome questions. Conceptual questions (not “How do you do problem 6?”). Rephrase poor questions into good ones.
- No evidence that small courses are better than large ones. Need extra high-quality preparation.
- Use TA’s, meet with them regularly. Grade horizontally (one TA per question?).
- Student ombudsmen (forward issues/complaints).

2.14 Review Sessions, et. al.

- Arguably more challenging: Respond to questions on any topic, need to think creatively on feet [DRC: students intense to pass exams, I’m much more exhausted after these sessions]. If novice: work all problems beforehand.
- Consider professor/grader observing TA help sessions.
- Possible responses: Do a similar problem. Get problem started. Outline steps.

2.15 On Being a TA

- If you are not self-directed, lifts some burden, but does not gives same experience as actually teaching.

2.16 Tutors

- Should one get a tutor? No. “*All learning of significant knowledge requires considerable effort on the part of the learner.*” [DRC: We provide tutors, but: students with tutors by far strongest correlation with low scores; “cooling out”.]
- Do not hire yourself out as tutor in courses you are teaching.

Chapter 3: Spiritual Matters

3.1 Breaking the Ice

- Start talking personally to students on first day.

3.2 Math Anxiety

- Real; refer student to professional. [DRC: Research is that only one-on-one counseling helps, nothing a teacher can do with a group in class.]
- Problem of some schools waiving math requirements due to this.

3. Inductive vs. Deductive Method

- Suggests working inductively from examples, giving some ideas but not proof. [DRC: Strongly disagree; best to share taste of discipline's professional tradition; get general rule in view as soon as possible. Working only examples delays, minimizes time considered, confuses students on the course goal.]

3.4 Who Is My Audience?

- Suggests (again) not teaching in traditional theorem-proof style [DRC: Disagree, as above; share taste of discipline, even with tiny one-line justifications called “proofs”; share the language used later].
- Possibility of other departments taking over calculus classes (specific and easier; KCC keeps them in-department). Teaching pays the bills. Do it well.

3.5 Mentors and Neophytes

- Teaching mentors are valuable. “Experience is a valuable commodity.” [DRC: Starting to see this with me & millennials; I have decades of experience with teaching, politics, games, pop culture, etc.]

3.6 Teaching Reformers

- Teaching reform & government funding (1986 on). Flipping lectures, cooperative learning, technology, etc. Should be considered. Needs serious guidance.
- “Talking mathematics is a high level skill... we should *plan to teach them [students] to do it.*”
- Personality types [DRC: All evidence is this makes no difference. Also: Rehab involves working the weak limb.] Learn different methods of discourse. [DRC: You need all of it.]
- Emotional debate; often, very different goals and concerns (body of math vs. student self-esteem vs. attrition rates).
- Harvard calculus project. Reform techniques are *harder* (more time & effort).

3.7 STEM

- Focus on STEM in Obama years 2008 on. Equity issues. Reports on “active learning” (must read, write, discuss, be engaged in problem solving... analysis, synthesis, evaluation) [DRC: Often in math we overlook lowest level skills].
- List of resources/organizations.

3.8 How to Ask, How to Answer

- Ask *specific questions* (not just “any questions?”).

- Quiz once a week generates inquiries. [DRC: I agree!]
- Turn weak questions into a better one. [DRC: Important, challenging skill! Inverting thought process to a question it *would* have been correct about.]
- Appendix ideas: Get students to blackboard; prepare oral reports; take turns making & grading quizzes; minute notes (jot down issue/concern); guest instructors; guest speakers; points for post-it note questions on tests; intentional mistakes for students to correct in lecture.

3.9 Inquiry-Based Learning

- Some theorists: Science class should model work of practicing scientists. Modified Moore method.

3.10 The Art of Discourse

- What does student take long-term? “(i) critical thinking skills and (ii) knowledge of and experience with discourse”. [DRC: See also Bain, etc.; teaching how to read & join conversation.] “The student is learning *different modes* of critical thought.” (Remember: Mostly math non-majors.)

3.11 Do I Have to Teach Calculus Again?

- Teaching is a team (department) effort, do a good job. Lower-division courses mostly pay the bills, and society expects a good job be done.

Chapter 4: The Electronic World

4.1 Teaching with the Internet

- Put stuff on the web. Caution against thinking it can substitute for classroom teaching.

4.2 Online Learning vs. Classroom Learning

- Reasons for in-class experience: Focus, removal from distractions, diversity, bonding/networking. Online consider D2L, Blackboard, Moodle.
- **Mature and disciplined can learn online; others not.**

4.3 MOOCs

- **Most don't finish MOOCs. [DRC: Experience with Udacity statistics abysmal. Experiments for remediation canceled in failure. Other experience considered unethical. Profit-seeking and alienation from our academic work. Counter: May get value if poor but very low-cost. Generally cooled off from bubble of 2013.]**

4.4 The Khan Academy

- Successful. Funded by Gates (as many others). High-school level [DRC: i.e., our level; I occasionally send students there for practice exercises. Some students know, others don't.]

4.5 The Flipped Classroom

- Videos pre-class, exercises in-class. Takes work to manage. Group work/assessment issues. [DRC: Was told only a few students do work beforehand, hoping for "critical mass" minimum. To me seems like just traditional reading/discussion expectation. Dislike videos.]

4.6 Computer Labs

- Software exercises. Bloom off since 1990's pricey and not always satisfying.

4.7 Clickers

- Some evidence instructors like more than students. Krantz impression: Better for non-math subjects.

4.8 Homework Solutions on the Internet

- Using Mathematica, Maple, MatLab; Wolfram Alpha, Sage, Maxima. Means likely credit can only be given for in-class tests. [DRC: I share this with my students to check work.]

4.9 Online Software

- Homework, testing, tutorials: WebWork, WebAssign, ALEKS, MathXL, etc. Mostly to automate homework grading. [DRC: WebWork tried, not available/supported at KCC. Other option by RSB. My Automatic-Algebra.org. Contact other CUNY school? **Research seems to say not a big help, but positive attitude.**]

4.10 The Course Web Page

- Documents, information, possibly discussions, etc. [DRC: Blackboard.]

4.11 Social Media

- **Facebook, Twitter, Instagram, Piazza, etc. A mistake to indicate students as “friends”. Proceed with caution. [DRC: Don’t like handing off data & other contacts to private advertising-based company. Have seen some rather unethical usage.]**

4.12 SmartBoards

- Krantz: prefer traditional chalkboard. [DRC: Don’t see any value in SmartBoards over simple slide show; haven’t seen anyone really use it. Standard slides can be distributed & dependably used on many systems.]

4.13 Reference URLs

Chapter 5: Difficult Matters

5.1 Non-Native English Speakers

- Non-native instructors should immerse in English to become reasonably good speakers. Be aware of cultural differences in American classrooms (students speaking up and questioning).

5.2 Late Work

- Drop two grades. Set policy in advance and use it consistently.

5.3 Cheating

- **Deal with it directly and firmly. Have a clear/consistent policy. Make announcement first day of class. Check university policies/limitations [DRC: At KCC need students agreement for academic penalty!]**
- Some places: signing honor code; would not work at institutions [DRC: Loathe this; students would be held anyway if not signed; inculcates mindless signing of forms.]
- Maintain control of tests, use large room, space out desks, check IDs, etc. Statistical checks. Photocopying test papers before return [DRC: I scan all mine; set up on one sheet each to facilitate this; also good for examples and reference later.]
- For plagiarism, check director, do not handle it alone.

5.4 Incompletes

- “I’ve given a lot of incompletes in my life and very few of them were ever completed.” Have uniform policy. [DRC: I only give this for passing work, missed final. Again: rare to have it made up; frequently students schedule dates and then don’t show up. Obviously harder to succeed after away from subject for some months.]

5.5 Discipline

- Stern response; or stop and get other students to shush them [DRC: This seems passive-aggressive. Can cause inter-student fracas.] Worst case: Cancel class for the day.
- Setting the tone early. Degenerate classes evolve over time. “Your attenae should be out for trouble.” Don’t use egregious tactics/disruptions for minor irritations (late student).

5.6 Mistakes in Class

- Be fully prepared for class. Constructive to let students point out corrections (hopefully minor). [DRC: I try to lavish this with praise.] If hopelessly lost, stop, apologize, write it up for next time. Frequently double-check & ask student for same.
- Don't skip steps. Write out every little detail.

5.7 Advice and Consent

- On non-academic questions, stick to things you know. Do not criticize other faculty when advising on other courses. Avoid very personal matters [DRC: send to school counselors.]
- Endless advising/help is detrimental to rest of academic career.

5.8 Sexism, Racism, Misogyny, and Related Problems

- Never physically touch students. Don't say or do sexist stuff (examples). Check guidelines. Example of job offer rescinded. Usually one word against another; allegations are easy and a black mark. Show respect, do not become defensive. Get help.

5.9 Begging and Pleading

- *"You must brush off these attempted bribes with the disdain that they deserve."*
- Younger faculty should consider dressing differently from students & maintaining distance [elsewhere: wants to be friendly with students]. Acting even mildly receptive to sexual advancements is courting disaster.
- Students commonly complain of success on homework, but choking on tests. They must practice until the skill is cold, with closed book. Most students have no idea how to study. [DRC: Do our students have space/time/literacy?]

Chapter 6: A New Beginning

6.1 The Role of the University Professor

- Transition from research-only to importance of teaching; it pays the bills. Teaching meshes nicely with research.

6.2 Closing Thoughts

- Like students, can suffer from cowardice and avoid the hard parts. [DRC: moral courage.] Teaching can be rewarding, useful, fun; it takes commitment and preparation.

Appendix

- Online: www.math.wustl.edu/~sk/teachapps.pdf